Module 3
Mathematics for gifted pupils

A note on terminology

The term used in these training materials is *able pupils*. Many teachers are currently more familiar with the broader term ‘able’ than with ‘gifted and talented’ which is now the DfES term for pupils whose ability is beyond conventional expectation. EiC uses ‘gifted and talented’, defined as the top 5–10% of pupils in a school. ‘Gifted’ pupils in an EiC cohort have latent or evident high ability in academic subjects, while ‘talented’ pupils have latent or evident high ability in a creative or expressive art or a sport.

Background to the module

This training module is intended to support mathematics departments considering their provision for able mathematicians. It presents a collaborative working session lasting approximately 75 minutes. This could be extended by allowing fuller discussion of the issues or by completing both options A and B. This module is designed to follow the introductory module 1, which considers whole-school issues related to the identification and teaching of more able pupils generally, but can be used independently of this. Suggestions about how a department might follow up this module are included.

You need to download and print all the materials.

The module contains:

- guidance for the head of department (or other presenter) on leading the session;
- slides which can be copied onto OHTs or projected from a computer;
- a guidance booklet and handouts which should be copied for all participants.

In working through the module, it is likely that departmental teams will identify issues they wish to pursue further. At several points, signposts to further guidance are included to help support such work. Publications referenced in the text are listed under resources in the guidance booklet.

Preparation and pre-module task

Before the session, participants should be given a copy of the booklet *Guidance on teaching able mathematicians* and asked to read it.
As presenter, you should familiarise yourself with the session notes and resources. You need to make a decision about which option (A or B), in the section headed ‘Planning for enrichment and extension’, you are going to use.

**Audience**

Mathematics departments in secondary and middle schools

**Objectives**

- To identify the characteristics of pupils who are able in mathematics
- To promote ways of using the *Framework for teaching mathematics: Years 7, 8 and 9* in planning to support and challenge able pupils
- To develop understanding of the booklet *Guidance on teaching able mathematicians*

**Resources**

OHTs 3.1–3.5

Handouts 3.1–3.4

For each participant:

- *Framework for teaching mathematics: Years 7, 8 and 9*
- the booklet *Guidance on teaching able mathematicians*
- the department’s medium-term plans or scheme of work for Years 7, 8 and 9
- (for option A) plain paper, a straight edge and pair of compasses
- (for option B) squared paper

**Module overview**

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<thead>
<tr>
<th>Pre-module reading</th>
<th>Read <em>Guidance on teaching able mathematicians</em></th>
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<tr>
<td><strong>Introduction</strong></td>
<td>15 minutes</td>
</tr>
<tr>
<td>Characteristics of able pupils</td>
<td></td>
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<tr>
<td><strong>Providing for able pupils</strong></td>
<td>10 minutes</td>
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<tr>
<td>Activity</td>
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<tr>
<td><strong>Planning for enrichment and extension</strong></td>
<td>25 minutes</td>
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<td>Option A or option B</td>
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<td><strong>Teaching strategies</strong></td>
<td>15 minutes</td>
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<td>Activity</td>
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<tr>
<td><strong>Next steps</strong></td>
<td>10 minutes</td>
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<tr>
<td>Departmental priorities for development</td>
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</table>
Introduction

Show OHT 3.1 and outline the objectives of the module.

<table>
<thead>
<tr>
<th>Objectives</th>
<th>OHT 3.1</th>
</tr>
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<tbody>
<tr>
<td>• To identify the characteristics of pupils who are able in mathematics</td>
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<tr>
<td>• To promote ways of using the <em>Framework for teaching mathematics: Years 7, 8 and 9</em> in planning to support and challenge able pupils</td>
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</tr>
<tr>
<td>• To develop understanding of the booklet <em>Guidance on teaching able mathematicians</em></td>
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</table>

Make these points.

• In the mathematics strand of the Key Stage 3 National Strategy, initial guidance on teaching mathematically able pupils is included in the *Framework for teaching mathematics: Years 7, 8 and 9* (see page 33 of section 1).

• Further guidance is presented with this module.

• A few very able pupils will need special consideration. This guidance is intended to help all schools address the needs of the most able 5%–10% in each cohort.

Give out handout 3.1, explaining that participants might like to make notes during the session to support their own professional development.

**OHT 3.2** shows some statements about mathematics for able pupils. Ask colleagues to discuss in pairs their attitudes to the statements.

<table>
<thead>
<tr>
<th>Able mathematicians in our school</th>
<th>OHT 3.2</th>
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<tbody>
<tr>
<td>Discuss the following statements.</td>
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<tr>
<td>• We all know who are the naturally able mathematicians in Year 7.</td>
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<tr>
<td>• We need to make sure we encourage and stretch the most able – they are the mathematics teachers of the future.</td>
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<tr>
<td>• Lots of our pupils get good GCSE grades, so we must be getting it right.</td>
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<tr>
<td>• Pupils who are good at mathematics are good at most subjects.</td>
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<tr>
<td>• We know what our able pupils think of their mathematics lessons.</td>
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</tbody>
</table>

After a few minutes share observations within the group. Keep the feedback and discussion brief. Most of the issues will be returned to during the session.
Make the following points where appropriate.

• All departments should have in place processes for identifying pupils with particular abilities and systems for sharing that information.

• There are, of course, many reasons for supporting the most able pupils but we all share a responsibility for encouraging more pupils to consider studying mathematics to higher levels.

• High attainment and high ability are not synonymous. Some potentially able mathematicians do not achieve highly at school.

• Aptitude in mathematics is not always reflected by high ability generally (and vice versa).

• Pupils’ attitudes to a subject and their lessons will have a significant impact on the extent to which they achieve highly.

Remind participants of the section in the guidance booklet on ‘Identifying able pupils in mathematics’ (page 1) and make the following points.

• Able pupils have individual profiles and few will demonstrate all of these characteristics.

• It is important to avoid stereotyping. Pupils may display talents in mathematics that would surprise teachers of other subjects.

• Pupils do not develop at a consistent pace.

Summarise some key messages.

• To ensure that pupils display their abilities we need to provide them with:
  – frequent opportunities to show high levels of aptitude;
  – systematic encouragement and praise for such displays;
  – rewarding responses (not just more of the same).

• Many of the characteristics that able pupils demonstrate relate to their ability to use and apply their mathematics. This should be an important focus when we review our schemes of work.

• Identification is not an end in itself. It is important, but it should not distract us from making appropriate provision the priority.

Further guidance on effective practice in identifying gifted and talented pupils is included in Providing for gifted and talented pupils: an evaluation of Excellence in Cities and other grant-funded programmes (Ofsted, December 2001; available from www.ofsted.gov.uk); see particularly paragraphs 30–40.
Providing for able pupils

Remind participants of the links between the Framework’s yearly teaching programmes and National Curriculum levels as listed under ‘Planning from objectives’ on page 3 of the guidance booklet.

Activity

Ask participants to read the paragraphs on able pupils on page 33 in section 1 of the Framework for teaching mathematics: Years 7, 8 and 9.

Point out the following.

• The key to providing for able pupils is to ensure through planning that they have plenty of opportunities to:
  – tackle challenging problems;
  – reinforce and deepen their understanding of standard topics;
  – experience mathematics as a rich and enjoyable subject to explore and enthuse about.

• It is not appropriate for able pupils just to be given more work to do of a similar nature to that done by others – they benefit from tasks that are different rather than merely longer.

Show OHT 3.3 which is based on the guidance booklet.

<table>
<thead>
<tr>
<th>Provision for able pupils</th>
<th>OHT 3.3</th>
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<tbody>
<tr>
<td>Able pupils need opportunities for:</td>
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<tr>
<td>• enrichment – applying skills and understanding to a wider range of problems, including unfamiliar contexts, and bringing together different strands of the subject;</td>
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<tr>
<td>• extension – working in greater depth, with increasing complexity, subtlety or abstraction;</td>
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<tr>
<td>• acceleration – provided by extending the ‘pitch’ of learning objectives to those expected of older pupils or introducing objectives from later years.</td>
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Make these points.

• Progression in mathematics is neither simple nor linear: it involves the development of a complex network of interrelated ideas.

• Access to these opportunities should vary according to the needs of individual pupils and to the mathematical topic being studied. Different pupils will benefit from enrichment, extension and/or acceleration at different times.
• While frequent opportunities for enrichment and extension should be built in to the teaching programme, the introduction of new objectives which the rest of the group will meet later (acceleration) requires careful monitoring to ensure progression in later terms.

• Constructive use of the objectives in the Framework will allow the needs of the majority of more able pupils to be met within the usual teaching programme.

A useful analysis of the issues related to the organisation of teaching groups is included in the QCA Guidance on teaching gifted and talented pupils under the heading ‘Inclusion’. This can be accessed via the QCA website or via www.nc.uk.net/gt/mathematics/inclusion.

Planning for enrichment and extension 25 minutes

Choose either option A or option B.

A Participants consider briefly three approaches to providing higher-level challenges. This may be more suitable in departments where differentiation within lessons is only partially developed.

B Participants consider one example in some detail, looking at the implications for choosing and presenting challenging tasks. This is likely to be more valuable where differentiation in classrooms is already well developed but the need to fully challenge the most able remains.

Option A

Distribute handout 3.2 and plain paper, a straight edge and pair of compasses to each participant.

Give participants 3 to 5 minutes to work through each of the tasks. After each one, take reactions to the questions posed. The notes below are intended to help focus the discussion. Throughout, consider if and where there are opportunities to incorporate the approaches illustrated in the department’s scheme of work.

Notes on the tasks

1 The simple adaptation of a routine closed question has opened it up so that pupils can explore it at their own level. Less able pupils will be encouraged to substitute their own values for \( a \) and \( b \) and work through the calculations until a pattern appears. The most able will be encouraged to generalise their solutions speedily and to justify their answers logically.

2 The construction is based on the fact that a line joining the mid-points of two sides is parallel to the third side. In this case, pupils need to draw on earlier learning (perhaps applying it in this context for the first time) in order to solve the problem. The level of construction is no more difficult but

...
the activity provides insights into the properties of triangles and a broader awareness of the possible uses of geometrical constructions.

3 The extension objectives and examples in the Framework will be particularly useful for teachers of able pupils in Year 9. The principle of using objectives from higher levels applies in Years 7 and 8 where objectives for Years 8 and 9 respectively will sometimes be appropriate. Clearly, their use for individuals within a group needs to be managed carefully to ensure progression in future years. Many objectives span several years; the Framework supplement of examples provides useful guidance on the development of concepts and skills across those years.

Option B

Distribute handout 3.3 and a sheet of squared paper to each participant. Handout 3.3 outlines part of a Year 8 lesson plan.

Ask participants in pairs to read through the sheet and quickly familiarise themselves with the main tasks, including the extension task for all.

When everyone is clear about what is involved, ask them to tackle the enrichment task.

After a few minutes, explain that whilst the enrichment task appears to be a relatively simple adaptation to the main problem, it requires some sophisticated thinking.

Encourage the group to debate the questions posed on OHT 3.4.

<table>
<thead>
<tr>
<th>Discussion questions</th>
<th>OHT 3.4</th>
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<tbody>
<tr>
<td>• Is the enrichment activity an appropriate challenge?</td>
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<tr>
<td>• What might we do to prepare the group who are going to tackle it to help them get into it?</td>
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<tr>
<td>• How might we change the problem to make it more accessible but still worthwhile?</td>
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Notes for presenters

• The appropriateness of the problem will depend upon the ability, experience and motivation of the group. However, many more able Year 8 pupils may be daunted by it as it stands.

• One way to ease pupils into the problem might be to revise properties of the diagonals of a square and suggest that this will help them find the missing corners as a first step.

• An interim (or alternative) challenge might be to consider squares with sides at 45° to the axes.
Conclude by making these summary points:

- Enrichment challenges need careful selection and presentation if they are to grab and maintain pupils’ interest.
- The problem should appear neither too trivial nor too daunting.
- Teachers need to spend time exploring such problems before their use in class.
- Departments can usefully collaborate on collecting good examples, with commentaries on their use, and building them into the scheme of work.

_The website www.1000problems.com is a useful source of challenging questions. It links questions to Framework objectives for each Key Stage 3 year group and each attainment target._

**Teaching strategies**

This section allows departments to begin to identify and share effective practice.

Show **OHT 3.5** which lists five classroom strategies for matching tasks to pupils’ abilities to ensure able pupils (and others) are appropriately challenged.

<table>
<thead>
<tr>
<th>Strategies to help match tasks to pupils’ abilities</th>
<th>OHT 3.5</th>
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<tbody>
<tr>
<td>• Start pupils on a task at an appropriate level of difficulty.</td>
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<tr>
<td>• Use challenging questions to extend thinking.</td>
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<tr>
<td>• Extend and open up tasks.</td>
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<tr>
<td>• Provide focused support as the teacher.</td>
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<tr>
<td>• Organise peer support and collaboration.</td>
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**Activity**

Ask participants to come up with specific examples of how they use each of these strategies. Collect the examples so that they can be recorded and shared within the department.

**Notes to help the discussion**

- It is useful for all pupils to take increasing responsibility for their learning. For example, where pupils are working through graded exercises for practice, some teachers encourage them to find the right level for themselves. The more able may be told: ‘Start at question 10; if it is too easy, jump to 15; if it is too hard, go back to number 5’. With encouragement, pupils can become adept at finding the questions which will challenge them.
Can we trust our pupils to work in this way?

• Effective teachers emphasise quality over quantity – tackling one challenging question is often more worthwhile than doing 20 routine ones.

Is this the message our pupils pick up?

• Most teachers recognise the value of extending the range of types of question they use in the classroom. Good questions for extending thinking include:
  – Why is this true?
  – When does this not work?
  – Can you see another way of doing that?
  – Can you think of a counter-example?
  – What would happen if …?
  – If the answer was … what might the question have been?

What others can we add to the list?

(Note: Further guidance on questioning can be found in the National Numeracy Strategy booklet Mathematical vocabulary which is available on the website www.standards.dfes.gov.uk/numeracy/publications.)

• Most topics can be extended by using one or more of the following approaches:
  – Can you make up some more questions like that?
  – Write a paragraph explaining the idea to someone else.
  – When might you use this method/idea/technique to solve a real-life problem? Collect some examples.
  – Go to the library and see what more you can find out about …

• For pupils who finish before their peers, occasionally involving them in a mentor or instructor role is valuable to them and the class:
  – Work with X and prepare to make a presentation to the class explaining …
  – Go and see if you can nudge Y who is stuck.

Consider producing a departmental aide-mémoire listing these sorts of generic strategy for team members to use with their lesson planning.

Suggest that each person identifies one strategy they do not currently use and plan to incorporate it into their teaching over the next week or two. Use handout 3.1 to record this. If possible, arrange to meet subsequently to evaluate each strategy’s effectiveness.
Next steps

Distribute copies of handout 3.4. This lists the features of effective teaching of gifted and talented pupils identified by Ofsted in their report *Providing for gifted and talented pupils: an evaluation of Excellence in Cities and other grant-funded programmes*.

Use this list to discuss how well established these features are within the department and identify which areas should be the focus for further development.

Conclude by agreeing a plan of action to take forward the department’s work in securing high quality provision for pupils who are, or have the potential to become, able mathematicians.

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